Trade and Growth in Africa
The Theoretical Framework of the TFED/ECA Global Macroeconometric Model

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Glossary of Symbols used

N indicates North; S indicates South, A Africa (EA East Africa; SA Southern Africa, NA North Africa and WC West and Central Africa) and OT indicates Other South (excluding Africa)

1 Debt Service ratio
\( \alpha \) output elasticity
\( t \) Mark-up rate in North (Price)
dAP, dAg, Output-capital ratio of the private & public sector, respectively
DBA Net commercial bank credit flow to Africa
DBS Net commercial bank credit flow to the South (=DBA+DBOT)
DBdpN North private sector demand for bank credit
DBOT Net commercial bank credit flow to other (non African) South
DBNS+pN Net commercial bank credit flow to South and North’s private sector
DDCN Northern government debt (flow)
DFA Foreign inflows (net of commercial bank lending and FDI) to Africa
DFAg Foreign inflow to the public sector of Africa
DFAp Foreign inflow to the private sector of Africa
DFOT Foreign inflows (net of commercial bank lending and FDI) to Other (non Africa) South
DFS Foreign inflows (net of commercial bank lending and FDI) to South
DJO, A,,OT, N Flow of saving deposit in international banks by OPEC, Africa, Other South and North
\( \eta \) an index of relative debt service ratio that shows solvency position
mS, Bank mark-up rate over world interest rate on Southern borrowers
\( b_N \) The labour output ratio in North
CN Real consumption in North
Cp Real private consumption in Africa
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>COILA</td>
<td>Real consumption of oil in Africa</td>
</tr>
<tr>
<td>COILN</td>
<td>Real consumption of oil in North</td>
</tr>
<tr>
<td>DCN</td>
<td>Northern government outstanding debt</td>
</tr>
<tr>
<td>DEFN</td>
<td>Deficit in North</td>
</tr>
<tr>
<td>Dev</td>
<td>Devaluation</td>
</tr>
<tr>
<td>Div</td>
<td>Diversification index</td>
</tr>
<tr>
<td>e</td>
<td>Nominal exchange rate (Africa)</td>
</tr>
<tr>
<td>DSR OT, A</td>
<td>Debt Service ratio of other South and Africa respectively</td>
</tr>
<tr>
<td>EXCAP</td>
<td>Excess capacity in North</td>
</tr>
<tr>
<td>EZ</td>
<td>Exchange rate fundamentals such as macroeconomic balance.</td>
</tr>
<tr>
<td>FA</td>
<td>Stock of Foreign inflows (net of commercial bank lending and FDI) to Africa</td>
</tr>
<tr>
<td>FDIS, FDIA, OT</td>
<td>Direct foreign investment, FDI to South, Africa, and Other South</td>
</tr>
<tr>
<td>FOT</td>
<td>Stock of Foreign inflows (net of commercial bank lending and FDI) to Other (non Africa) South</td>
</tr>
<tr>
<td>fpS</td>
<td>Factor payments for South</td>
</tr>
<tr>
<td>FS</td>
<td>Stock of Foreign inflows (net of commercial bank lending and FDI) to South</td>
</tr>
<tr>
<td>GA</td>
<td>Government consumption expenditure in Africa</td>
</tr>
<tr>
<td>GDPmi</td>
<td>GDP in mining sector</td>
</tr>
<tr>
<td>GN</td>
<td>Government expenditure in North</td>
</tr>
<tr>
<td>HdNi</td>
<td>Commodity Stock demand (North)</td>
</tr>
<tr>
<td>HsNi</td>
<td>Commodity Stock supply (North)</td>
</tr>
<tr>
<td>IA, Ag, Ap</td>
<td>Real investment in Africa, Africa public sector and private sector, respectively</td>
</tr>
<tr>
<td>IN</td>
<td>Real investment in North</td>
</tr>
<tr>
<td>IS</td>
<td>Real investment in South</td>
</tr>
</tbody>
</table>
QS Real GDP in South

iw World interest rate

iw* World interest rate on concessional

Inst Institutional condition indicators

JO, A,,OT, N Saving deposit in international banks by OPEC, Africa Other South and North

KA Real capital stock in Africa

LA Labour

Macro Macroeconomic environment

MInOT Mining value added in Other South

MMfgOPE, A. Real import of manufactured goods by OPEC and Africa , respectively.

MN, MOT MA Import of North and other South respectively

mNoil Import propensity of oil consumption in North

mNs Import propensity of oil consumption in South

mopecN import propensity of OPEC for Northern goods

MOoilN, OT Real import of Oil by North, and Other South (OT) respectively

MR S Ni Import of primary commodities by North from Africa i= 1...4

MUP Manufactured unit price index

othDSi Other debt service component (discrepancy)

otGRA Other government revenue

otMN, otMOPEC Other imports of North, and OPEC, respectively

otXN, otXA Other exports of North, and Africa

Pcoal Price of coal

PMfgN Price of Manufactured goods of North which could be a proxy for PQN

Poil Price of oil

Politics a dummy, which has a value of 1 when a country is conflict-prone

(used as a proxy for political or governance problems) and zero otherwise,

PQA Domestic price in Africa

PQN Domestic price in the North which could be approximated by PMfgN

PRi viii Export price of primary commodities from Africa

Prin Ai Principal payment by Africa
QN, QOT, QA  Real output in North, other South and Africa, respectively
QOILN  Real output of oil in North
Real growth rate of African output
Real aggregate demand in Africa
rA  Real interest rate, nominal interest less inflation
( growth of PQ), (in Africa)
RER  Real exchange rate
Rimit  Remittance
SAg, SAp  Saving of Africa, public and private, respectively
SN, SA, sg, sp  Saving of North & Africa (total, public and private), respectively
tN  Tax rate in North
TA, Td, iex A  Total revenue, Direct and indirect tax revenue, respectively in Africa
TFP (A)  Total Factor Productivity or the Solow residual
ToT  Terms of Trade which is defined as PRi/PQA
wN  Wage rate in North
Xss Si  Real global supply of primary commodities from South
XN  Export of the North
XA,  Real Export of Africa (total)
XOilA,  Real export of oil by Africa (total)
XOilNA, XOilWC  Real export of oil by North Africa and West and Central Africa, receptively
XR Ai, XR OTi  Real export Supply of primary commodities from Africa, and Other South, respectively.
Xss Si  Global demand for primary commodities from South
Yp OT, A  Real percapita income of other South and Africa respectively
ZAg  is the sum of, ZB and, Zpr Total resource transfer from the banking and the private section to the public sector in Africa
ZB  Public borrowing from banks
Zpr  Resource transfer from private to the public sector
Zf  Financial sector development

The Trade, Finance and Economic Development (TFED) of the Economic Commission for Africa is in the process of developing a global econometric model for Africa. This model could be used for forecasting major macroeconomic trends in Africa and preparation of economic outlook about Africa, for analysing global and African trade, finance issues, as well as analysis of macroeconomic policies and external shocks and their implication on Africa. This paper is the theoretical framework of the TFED/ECA’s global macro model that we referred as AFRIMOD (African Global Macroeconometric Models). The model is similar to other applied global models such as IMF’s Multimod (Multi-country Econometric Model) and the World Bank’s Bank-GEM (Bank General Equilibrium Model), among others.

The specification of this African based global (or North-South) model outlined below is based on the theoretical North-South models of Taylor (1981, 1983, 1991), Darity and FitzGerald (1982), Kanbur and Vines (1986), Murshed (1990) and Vos (1994) as well as the theoretical-cum-empirical models of Marquez and Pauly (1987), Masson et al (1990) and Alemayehu (2002). In relation to the theoretical underpinning, the basic characteristic of the model is its effort to combine a Kaleckian/Keynesian North, a Lewis type economy for the South, and an oil producing and supplying region (representing the OPEC countries). The North and OPEC will be dealt with in a very limited fashion, with an attempt to develop and elaborate upon the modelling of Africa taken as the prime objective. This approach stands in opposition to the current practice in North-South modelling, in which the focus has, mainly, been on elaborating upon the section of the model dealing with the North. All parameters, with the exception of some ratios, could be econometrically estimated. This means that the model may also be interpreted as an empirical model. In what follows an elaborated discussion of this model is done. The discussion will be organized as follows. In sections 2.1 to 2.5, the model is specified. Although the specification set out below takes a linear form, all estimations could be undertaken using a log-log model. However, in order to ensure compatibility with the identities specified in the model, appropriate conversion of the log-log model is required. Model parameters specified in this study are also assumed to be positive.

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II. The North: The Industrialized Countries

The modelling of the North, adopted in this study, is not an elaborated one. There are two main reasons for this. First, the focus of the study is mainly on the modelling of the South, and Africa in particular. Second, if need be, an elaborated model of the South, developed in this study, may be combined with the existing models such as the IMF’s Multimod, the World Bank’s Bank-GEM or the OECD’s Interlink, which have an elaborated North. Notwithstanding the existence of a number of models incorporating an elaborated North, we have specified, below, a less elaborated model of North, based on Taylor (1981). The use of this, less elaborated, model for the North has three desirable consequences. Firstly, it allows one to specify the North along theoretical lines, which are best suited to the purposes of this study. Secondly, incorporation of this, less well-elaborated, model for the North allows one to understand the relevant interaction of the two regions and, hence, subsequent feedback effects of the North on the South. Third, this formulation allows an easy link with other global models that have an elaborated model of the North.

2.1. The Major Macro Aggregates Sub-bloc

The North is assumed to be an economy with excess capacity, with output taken as being demand determined, along Keynesian/Kaleckian lines. This is given by Eq.1, which is the aggregate equilibrium condition, and the basic closure of the model for the North.

\[ P_a Q_N = P_a C_N + P_a I_N + G_N + X_N - M_N \]  

[1]

The demand component comprises a simple consumption function that depends on income (Eq. 2), an investment function, which depends on income and world interest rate (Eq. 3) and exports and imports (given in equations 4 and 5, respectively). By making the two regions, OPEC and the South, part of the destination and source for exports and imports of the North, respectively, equations 4 and 5 ensure a trade linkage among the three regions.

\[ C_N = a_2 + b_2 Q_N \]  

[2]

\[ I_N = a_3 + b_3 Q_N - c_3 i_W - d_4 EXCAP \]  

[3]
The Northern (manufactured) price equation (Eq. 6) is based on a Kaleckian mark-up pricing approach, which assumes an oligopolistic market structure in the North. The specification follows the works of Taylor (1981, 1983, 1991), Darity and FitzGerald (1982), Marquez and Pauly (1987) and Vos (1994). It is positively related to prices of intermediate inputs, comprising wages, imported raw materials and oil price, and inversely related to excess capacity in the North.

\[
X_N = M^k_N (M^k_{OPEC} + M^k_A + M^k_{OTD}) + otx_N
\]  

\[
M_N = R^S_N (P_R) + P_{oil} (M^{Oil}_N + XOil_A + XOil_{W}) + otM_N
\]

The Northern (manufactured) price equation (Eq. 6) is based on a Kaleckian mark-up pricing approach, which assumes an oligopolistic market structure in the North. The specification follows the works of Taylor (1981, 1983, 1991), Darity and FitzGerald (1982), Marquez and Pauly (1987) and Vos (1994).² It is positively related to prices of intermediate inputs, comprising wages, imported raw materials and oil price, and inversely related to excess capacity in the North.

\[
PM^k_N = a_6 + b_6 P_{oil} + c_6 w_N + d_6 P_R - e_6 EXCAP
\]

Following a Kaleckian/Kaldorian tradition, private saving is assumed to come from profit and non-wage income, which, in turn, depends on the mark-up pricing scheme specified in Eq. 6. Following Murshed, the level of saving is further augmented by factor income from abroad as well as interest income from Southern debt. Since such income usually accrues to multinational firms and banks, their propensity to consume out of these factor incomes is assumed to be negligible. This is given in Eq. 7.

\[
S_N = s_N [w b_N + P_{oil} m_{oil} X_N D_{N} N] + (1 + \mu_S) i_w B_S + i_w F_S + i_w (D_N - J_N - J_{\phi}) + \hat{p}_S
\]

Factor payment receipts of the North from the South is formulated based on a historical trend of factor payments and foreign direct investment (FDI) to South. Thus, the coefficients in Eq. 8, below, are average ratios of factor payments to FDI, for the period under analysis.

\[
\hat{p}_S = a_8 FDI_{\phi} + b_8 FDI_A + c_8 FDI_{W} + d_8 FDI_{H} + e_8 FDI_{G}
\]

². The pricing rule in [6] could also be written as \( M^k_N = (1 + g) [P_{oil} + w_N + P_R] \) where \( g \) is the mark-up rate that depends on the degree of monopoly as argued by Kalecki.
The North also interacts with the OPEC (oil producing and exporting countries), through its oil consumption function, which is given in Eq. 9, as well as subsequent demand for oil imports, which is given by Eq. 10. Real consumption of oil in North is related to price of oil, the price of a substitute, such as coal, and the level of economic activity in North.

\[ C_{OIL_N} = a_{9} - b_{9}P_{oil} + c_{9}P_{coal} + d_{9}Q_{N} \]  \[ [9] \]

\[ M_{oil}^N = (C_{oil}^N - Q_{oil}^N) - W_{N} - W_{N} \]  \[ [10] \]

2.2 The Official Capital Flows and FDI Sub-bloc

2.1.2 The Official Capital Flows Sub-bloc

Following Vos (1994), the deficit of the North may be specified as in Eq. 11, where it is financed by issuing bonds, or by assuming that the government in the North is a borrower, having preferential access to international capital markets.

\[ DEF_{N} = \Delta D_{N} = G_{N} + i_{w}D_{N} + \Delta F_{S} - t_{N}(P_{N}Q_{N}) \]  \[ [11] \]

However, the Vos version of this is extremely aggregated and does not specify how inflows to the South are determined and allocated. Hence, a major extension of Vos’ approach is undertaken by further specifying official capital flows to the South, using equations 12 & 13. Given the total inflow to the South (\( \Delta FS \) in equation 11) as a policy variable in North, or determined by North’s affordability criteria, the allocation of the total inflow, across the South, given by Eq.12, is based on the theoretical approaches of, inter alia, Ruttan (1992), OECD (1985), Mikesell (1968), Mosley (1987), Dudley and Montmarquette (1976) and McGillivary and White (1993). This literature emphasises economic, strategic and political self-interest, as well as developmental/humanitarian considerations (see Alemayehu 2002). Of these, economic considerations are represented by the first two arguments within equation 12, strategic and political self-interest by the third argument and developmental/humanitarian considerations by the fourth argument. This specification is extended to include indicators of relative repayment capacity (represented by the fifth argument in Eq. 12), since this represents another important economic variable not considered in the literature. The estimation of Eq. 12, using an error correction model (ECM) allows, through the error correction term, to capture not only the impact of past budgetary practices but also previous historical/colonial ties.
\[
\Delta F_A = a_2 + b_2 \left( \frac{M_A}{M_\phi} \right) + c_2 \left( \frac{FDI_A}{FDI_\phi} \right) + d_2 \ Politics + e_2 \left( \frac{Y^p_A}{Y^p_\phi} \right) + f_2 \left( \frac{DSR_A}{DSR_\phi} \right)
\]

2.1.3 The Foreign Direct Investment Sub-bloc

Equation 13 is specified using a simple ‘eclectic’ approach to the determination of Foreign Direct Investment (Dunning, 1993) and theory of industrial organization (Helleiner, 1989). The first argument in this equation shows market size, the second is used to capture the concentration of FDI in the mining sectors of most African countries, the third argument is a dummy, which indicates whether the country is conflict-prone (used as a proxy for political or governance problems) or otherwise, and finally, the lagged dependent variable is used to capture other historical considerations, as well as ‘economies of specialization’ or internalisation used in pursuing FDI. In the first two arguments the variables are computed relative to other South, in order to indicate the choice faced by Northern agents.

\[
FDI_A = a_3 + b_3 \left( \frac{Q_A}{Q_\phi} \right) + c_3 \left( \frac{MIN_A}{MIN_\phi} \right) + d_3 \ Politics + e_3 \left( FDI_A \right)_{t-1}
\]

Both equations 12 and 13 are fitted to the three regions, which form the focus of this study.

2.3 The Commodity Sub-bloc

The commodity sub-bloc arises out of the empirical analysis, presented in Alemayehu (2002). Ideally the modeling of the commodity market could have been undertaken along the lines of Hwa (1985) and Ramanujam and Vines (1990), both of which are based on the theoretical work of Adams and Behrman (1976). However, this could not be carried out due to the absence of available data on commodity stock holding by economic agents in developed countries. As discussed in Alemayehu (2002), the commodity bloc, specified in Eqs.14 and 15, indirectly addresses the stock holding behavior of Northern agents. Following an econometric analysis, the equilibrium version is found to be relatively the better and, hence, this version is given below.

---

3. See Alemayehu (1995) for a discussion of some theoretical implications of the incorporation of such models into a Global Model of Africa.
Noting that all equations are set in log-log terms, we can re-write equation [14] in its original form and solve it for PRi. By taking the log of the result, equation [14] could be written as Eq. 14a.

\[ X_i^{d} = a_i + b_i \frac{Q_N}{\Phi_N} - c_i i_w - d_i \left( \frac{R_i}{\Phi_N} \right) \]  \[ [14] \]

Equations 14 and 15 are used for four commodity categories (i=1...4) consisting of food, beverage, agricultural raw material, and, finally, metals, ores and minerals. This distinction is found to be empirically important (see Alemayehu 2002, Bond 1987). Thus, through Eq. 16, the global commodity market for each commodity category clears for price. This is in line with various studies relating to the functioning of commodity markets, starting from the classic works of Prebisch (1962) and Singer (1950). Following a lag period, the price derived in this market, in turn, determines the supply of exports from Africa (Eq. 33). Moreover, through the identity given in Eq. 50, supply from Africa has an effect on global supply, which is given in Eq. 15. This is because we imposed the condition that \( X_i^{s} = K_{\delta} + X_{OTi}^{s} \). Thus, the global commodity market and African supply are dynamically linked.
III. The South: Africa

3.1 The Major Macro Aggregate Sub-bloc

Within this sub-bloc, the South is divided into two regions, comprising Other South (OT) and Africa (A). The latter, in turn, is further divided into four sub-regions, based on the UNECA classification. These are East Africa (EA), Southern Africa (SA), North Africa (NA) and West & Central Africa (WCA). An historic and economic justification for such a classification, based on the nature of each region’s interaction with Western Europe, may be found in Nzula et al (1979) and Amin (1972). The Model specified below is a prototype model used for each region. The rest of the world, excluding the North, and Other South categories are, on the whole, assumed to be exogenous to the model, the major features of which are outlined below (see Appendix I for country classification).

Unlike the North, the South is assumed to be a supply constrained economy. In general, the long-run supply-constrained nature of such economies may be depicted by a production structure characterized by full capacity utilization (Taylor, 1981, 1983). However, in Africa, the past three decades have witnessed some vital sectors of the economy being faced with under capacity utilisation, arising from problems relating to the supply of imported inputs. This has been especially true of manufacturing, which is dependent on intermediate imports, as well as those sections of the agricultural sector, which are dependent on imports of fertilizer, and transport facilities. This theme has been taken up in the ‘import compression’ literature, in general, and by Ndulu (1986, 1991) and Rattso (1992a), in particular. We have explicitly incorporated these considerations in modelling the South, through equations 19, 20 and 21. All type of foreign inflows, except foreign direct investment (FDI), are specified as determining the level of imports. We, however, began by the specification of the production function as well as the demand for factor inputs given by equations 17 to 19.

Equations 17 to 23, together summarise the major macro bloc of Africa. Beginning with Eq. 17, this equation illustrates the supply of output from the South.

\[ Q_A = A \left( \beta L_A^{-\alpha} + \beta K_A^{-\alpha} \right)^{\frac{1}{\alpha}} \]  

[17]

4. However, the Sudan is excluded from NA and included in the ESA region, since its economic structure is similar to the latter.
The level of output (Supply from Africa) (QA) is assumed to come from a CES production function with capital that may include land, hoes or animal power (KA) and labour (LA). Rainfall (R) and other productivity enhancing inputs such as fertilizer (F) and imports (M) are considered as arguments at estimation stage by adding them on ad hoc basis. The function in equation [17] is specified assuming an optimizing self employed household and/or firm that is endowed with K and L (see Alemayehu and Huizinga 2004).

We relate the African supply to the issue of trade and diversification of African exports by modifying the CES production function above to accommodate this concern. This is done along the similar specifications that relates trade issue as part of technology models of the CES production function such as Rivera-Batiz and Romer (1992); as well as the endogenous growth model that build on the ‘learning by doing’ model of Arrow (1962) and the related ‘knowledge spillover’ hypothesis of Romer (1986). The simplest version of this is what is called the AK (the ‘linear-in-K) model (See Barrow and Sala-i-Martin 2004; Valdes 1999, Jones 1998). Such endogenous growth models could have a CES formulation (see Barro and Sala-i-Martin 2004: 63-71).

Based on work done on diversification issue in Africa (see Ben-Hammuda, et al 2006), we assumed that ‘learning-by-doing’ and ‘knowledge spillover’ would be captured through enhanced trade (hence global integration) and export diversification in African context. Diversification (Div) together with other factors such as spending on research and development (R&D) and other exogenous factors such as conflict, rainfall and financial sector development (Zf) are assumed to affect the technology parameter (the A or TFP) as specified in equation 17(a).

\[ A(TFP) = a_{\alpha} + b_{\alpha} Div + c_{\alpha} R \& D + d_{\alpha} Zf \]  \[17a\]

---

5. The CES production function could be estimated at two stages assuming \( \alpha \beta L = (1 - \alpha K) \) (see Marquez and Pauly, 1989: 94-95). Alternatively, a linear approximation of this using either Klementa’s first order Taylor approximation or Jorgenson and Lau’s second order Taylor approximation yields an estimable version whose general representation could be given by a translog function of the following form

\[ \log Q_A = \log A + \alpha_k \log K + \alpha_L \log L + \alpha_k [\log K]^2 + \alpha_L [\log L]^2 + \alpha_k \log K \log L \]

where the hypothesis of CES could be tested by testing the restriction \( \alpha_K = \alpha_L = -\frac{1}{2} \alpha_K \) (see Thomas 1993: 326-331).
Diversification in turn is assumed to be determined by level of income (QA), macroeconomic environment (Macro) and institutional variables (Inst) as specified in equation 17(b)

\[
Div = a_{\gamma} b + b_{\gamma} Q_A + c_{\gamma} Macro + d_{\gamma} Inst \tag{17b}
\]

Coming to the two factor inputs, the supply of labour is assumed exogenous in the model (determined by population growth). The demand for labour (and hence the number of employees) is derived from the profit maximizing condition using the CES production function given in equation 17. The optimal labour input according to this CES production function is:

\[
L_A^d = A^{-\alpha/\sigma} \left( \beta_L \right)^\sigma \left( Q_A \right)^{1-\sigma} \left( \frac{w}{p_\theta} \right)^{\sigma} \quad \text{where} \quad \sigma = \frac{1}{1+\alpha} \tag{18}
\]

Total employment growth equals the growth of the economically active population. In the short run we assume there is no migration between the urban and rural sectors. Employment in rural and urban sectors both grow with total employment.

The other factor input, the investment function, is derived from the conditions for an optimal level of capital stock (K) with Pk as price of capital goods. The optimal macro capital stock is given by:

\[
K = A^{-\alpha/\sigma} \left( \beta_k \right)^\sigma Q_A \left( \frac{p_k}{p_\theta} \right)^{\sigma} \tag{19}
\]

From this we get:

\[
\frac{\Delta K_K}{K_{-1}} = \frac{I_{-1}}{K_{-1}} = \dot{Q}_A - \sigma (\dot{p}_k - \dot{p}_\theta) + \sigma \tag{19a}
\]

It is sometimes argued that profits (\(\pi\)) allow internal financing of investment, which is cheaper than external financing. In addition, the capacity utilization rate (q) may play a role as a direct indicator of the difference between optimal and actual capacity. Similarly, public investment (Ig) might have also a crowding-in or crowing-out effect in most African countries. Adding these elements, we get:
\[
\frac{\Delta K_{p k}}{K_{-t}} = \frac{I_{\theta}}{K_{-t}} = \hat{Q}_A - \sigma (\hat{p}_k - \hat{p}_\theta ) + \sigma
\]

\[
= \hat{Q}_A - \sigma (\hat{p}_k - \hat{p}_\theta ) - \sigma \frac{d}{r + \delta + \sigma} + \alpha \left( \frac{\pi}{k} \right)_{t-1} + \mu \hat{q}_\theta - \hat{J} + \eta \hat{q}_\theta + b \hat{M}
\]

[19b]

Where we used the fact that \( pk = (r+d+r^-)p_i = P_{mfg} \). Capital is often considered a quasi fixed factor of production, because changing the capital stock takes a lot of time and involves important adjustment cost. Therefore, lags may be important in this equation. For the same reason, expectations matter as well, although they are difficult to model, and empirical models of investment with various expectations terms have not been usually very successful (see Alemayehu and Huizinga 2004).

The investment equation above is linked to the level of imports as defined in equations 20. As in Alemayehu (2002) as well as in FitzGerald et al (1992) the investment functions are based on a theoretical framework that depicts the externally constrained nature of investment in developing countries. Given the flow of foreign finance (whether based on trade, external borrowing or remittance), as well as outflows of foreign exchange (with debt service payment included), the level of imports is defined (by way of Eq. 20) as an accommodating variable. The level of reserves is also assumed to be zero, which is not an unreasonable assumption, given the precarious nature of reserves in Africa. A further simplifying assumption, which is employed, is that (after settling external financial obligations, including capital flight), all types of aid as well as remittances are best considered as representing an additional source of external finance, which will enhance the level of imports. Eq. 20 is also central in incorporating the basic themes, summarised in the import compression literature.

\[
M_A = X_A + \Delta F_{A} + \Delta B_{A} + FDI_{A} + i_w J_S + Rimt - \Delta J_A - (1 + \mu) i_w B_A - i_{w_s} F_A - k_8 FDI
\]

[20]

Where \( k_8 = b, c, d \) and \( e \) as specified in equation 8

Consumption (real) is determined by a simple of two periods intertemporal optimization. In reduced form this is given as a function of real disposable income, real interest rate the household’s real wealth. The foreign inflow and remittances are added on ad hock bases to capture the possible impact
of these influential sources of income in African set-up.\footnote{This simple equation can be derived from the idea of a representative consumer that maximize the following two period intertemporal problem:}

\[
C_{PA} = a_3 + b_2 (Q_A - T_A) + C_2 (r_A) + +d_2 W_A + e_2 (\Delta F_A + Rimt)
\]  

\[
C_{oil,A} = a_3 + b_2 P_{oil} + c_1 (\Delta F_A + Rimt) + d_2 Q_A
\]  

Equation 21b summarizes real consumption level of oil by the South. However, unlike Marquez and Pauly (1987), in which oil consumption of the South is considered to be a substitute for capital, in the context of cost minimization, we argue here for its complementary. Thus, it has a relatively rigid (inelastic) demand that depends on its own price, availability of external finance and the South’s level of output. The econometric estimation would also appear to lend support to this latter position (see Alemayehu 2002).

\[
M^{\frac{k}{A}} = a_2 + b_2 M^{\frac{k}{N}} + c_2 Q_A + d_2 (\Delta F_A + Rimt)
\]

\[
XOil_A = a_3 + b_3 P_{oil} + c_3 FDI_{A_t-1} + d_3 Q_N
\]

---

\[
\argmax(c_1,c_2) : \log c_1 + \frac{1}{1+\delta} \log c_2
\]

subject to: 

\[
\frac{c_1}{1+r} c_2 = y^d_1 + \frac{1}{1+r} y^d_2 + wealth_0
\]

Where $c_i$ and $y_{id}$ denote real consumption and real disposable income in period $i$, for $i = 1, 2$. The superscript $e$ denotes expected value. $r$ is the real interest rate and $d$ the personal discount rate. $wealth_0$ denotes wealth accumulated from the past. The maximand is the present discounted value of intertemporal utility. Under the assumption that the personal discount rate $d$ is equal to the real interest rate $r$, the first order conditions for this problem imply $c_1 = c_2$ which shows the basic idea of consumption smoothing (see Alemayehu and Huizinga, 2004 for detail and extensions).
Equation 22 gives manufactured imports of South, which is specified as a function of own price, domestic demand and availability of foreign exchange. The export of oil function, specified in equation 23 is not used in relation to the ESA region, since that region does not produce oil. Exports of oil are assumed to depend on the relative (to coal) price of oil, economic activity in the North and supply inducing factors such as FDI (which is important as a proxy for technology of oil exploration which invariably comes from the North). Since the econometric analysis did not produce a statistically significant result in relation to the price of coal and Northern economic activity, they are omitted from equation 23. The importance of supply factors in oil export functions is discussed in Oshikoya’s (1989) model for Nigeria. He uses output of oil as a regressor. However, since the relationship between output and exports are proportional, this is tantamount to regressing a variable on itself. Thus, in order to rectify this shortcoming, lagged (domestic) investment in the sector is used, in this model. The latter, however, turns out to be statistically insignificant while FDI remains significant (see Alemayehu 2002). Hence, this serves as additional justification for the inclusion of FDI in Eq. 23.

3.2 The Fiscal Response Sub-bloc

The fiscal response of the South to external finance is modelled, in nominal terms, using equations 24 to 32. Government fiscal response in most African countries is assumed to be influenced by an uncertain external economic environment (particularly in relation to foreign inflows) as well as the political cost of deficit (inflationary) financing. Tax revenue is modelled using equations 24 (which deals with direct tax) and 25 (dealing with indirect taxes). Government expenditure and borrowing, as well as the relevant identities, for this sub-bloc are modelled using equations 27 to 32, where $Z_{Ag}$ represents the sum of government borrowing from banks ($Z_b$) and resource transfers from the private sector ($Z_{pr}$) (See Alemayehu et al 1992; FitzGerald 1993). We note from equation 32 that the private sector investment is implicitly assumed to accommodate to any financing problem, say through a decline in investment.

$$T_{Ad} = a_{24} + b_{24}Q_{A} + c_{24}ΔF_{A}$$  \[24\]

$$T_{Aex} = a_{25} + b_{25}C_{p} + c_{25}(X + M)_{A} + d_{25}ΔF_{A}$$  \[25\]

$$T_{A} = T_{Ad} + T_{Aex} + otGR_{A}$$  \[26\]

$$G_{A} = a_{27} + b_{27}T_{A} + c_{27}ΔF_{A} + d_{27}G_{A} (-1)$$  \[27\]
3.3 The Commodity Export Supply Sub-bloc

The supply of exports from Africa is modelled based on the analysis presented in Alemayhu (2002). As described in the study, the real exchange rate based model fits the data best and, hence, it is this version of the model which is employed here. The ECM version of the export supply equation (Eq. 33) shows both the short run and a long-run impact of price and capital formation indicators on supply of commodities. The short-run parts of the argument are based on (latent) capacity utilization theory (Wickens and Greenfield 1973, Goldestin and Khan 1978, Chu and Morrison 1986, and Hwa 1985). The last argument in Eq. 33 is used to indicate the impact of capital formation (investment) or lack thereof in the sector. The lag structure (i.e., k) is five to six years when i is beverage or minerals while one to two years for food and agricultural raw materials.

\[
S_{Ag} = T_A + otGR_A - G_A \tag{28}
\]

\[
S_{Ap} = a_{29} + b_{29} Q_A \tag{29}
\]

\[
S_A = S_{Ag} + S_{Ap} \tag{30}
\]

\[
\Delta F_A = \Delta F_{Ag} + \Delta F_{Ap} \tag{31}
\]

\[
Z_{\Delta} = Z_B + Z_p = I_{\Delta} - S_{\Delta} - \Delta F_{\Delta} \tag{32}
\]

3.4 The ‘Dutch Disease’ Sub-bloc

The interaction between Equations 33, 34 and 35 describes a ‘Dutch Disease’ effect. The literature suggests that foreign inflows could lead to an appreciation of the exchange rate, through the demand that it might create in the non-traded sector. Indeed, the latter could further affect the performance of the traded sector, through equation 33 above. Thus, the estimable version reported here is based on the reduced form equations derived from the broader framework given in Edwards (1989) (see
Alemayehu 2002 for detail). However, in order to make the specification consistent within the North-South model specified in this study, the following two steps are followed.

First, in equation 34, domestic price is set as a function of excess demand and other determinants of price that included foreign inflow that is assumed in the Dutch Disease literature to affect the price of none-tradables. For economies with an important food sector, Taylor (1983) has described the formation of prices using excess demand. Within a national macroeconomic framework, and when data permits, the disaggregation of excess demand into food and non-food sectors has quite profound implications (See Taylor, 1983: 39-48). However, in this study, an aggregate version of a similar equation is used. This equation has also incorporates the possibility of imported inflation, or a mark-up pricing effect (see Eq. 34). Moreover, by using an ECM formulation, price stickiness is also assumed. In the second stage, we have specified determinants of exchange rate which in turn is linked to the determinants of equilibrium exchange rate given by equation 35a as specified by Edwards (1989) (see also Alemayehu 2002; Israd 1995). It is also linked with the determination of price of primary commodities as given by Eq.14.1 Given the two equations, the real exchange rate is given by the definition of the real exchange rate as given by equation 35a that links equations 34, 35a and 14.1.

\[
P_{Q_A} = a_3 + b_4 \Delta F_A + c_3 MUP + d_3 (Q_A^d - Q_A) \tag{34}
\]

\[
e_A = a_3 + b_3 \hat{Q}_A + c_3 G_A + d_3 \text{Dev} + e_3 (ToT) + f_3 (\Delta F_A) + g_3 e_{A_{t-1}} \tag{35a}
\]

Where: \( a_3 \) is GDP growth assumed to show technological change; \( G \) government consumption expenditure assumed to show governments spending on the non-traded sector; \( \text{Dev} \) is nominal devaluation; \( \text{ToT} = PRi/PQA \) is terms of trade; \( \Delta F \) foreign inflow. The constant term \((a35)\) captures, among other things, the impact of the variation of macroeconomic policy from its sustainable level.

\[
RER_{Ai} = \frac{P_{Ri}}{P_{Q_i}} \quad i = 1..4, \text{ for the four commodity categories}
\]
IV. Major Oil Exporting Countries (OPEC)

The total output of OPEC is assumed to depend on demand for oil both in the North and South which, in turn, depends on the level of output for each region, as shown in equation 36.

\[
Q_{OPEC} = m_N Q_N + m_S Q_S
\]  

[36]

Given the import of oil by the North (Eq. 10) and that of Africa (Eq. 21) from OPEC, oil imports of the rest of the South are derived as residual using Eq. 37. Although imports and consumption of oil may safely be assumed to be equal in the EA and SA region and the majority of countries in WCA (Eq. 21), this assumption does not hold for NA and the major oil producers of WCA (Gabon, Cameroon and Nigeria). Thus, oil consumption for non-oil producing countries actually equals consumption of imported oil.

\[
\mathbf{M}^{\text{Oil}} = Q_{OPEC} - \mathbf{M}_N^{\text{Oil}} - C_{oil_A}
\]  

[37]

\[
\mathbf{M}^{\text{g}}_{OPEC} = m_{OPEC_g} Q_{OPEC}
\]  

[38]

\[
\Delta J_{OPEC} = P_{oil} Q_{OPEC} - (\mathbf{M}_N^{\text{g}} \cdot \mathbf{M}_{OPEC}^{\text{g}}) - otM_{OPEC} + i_w J_O
\]  

[39]

Finally, equation 38 summarises the portion of OPEC’s income that is spent on Northern goods, with the remainder, including interest income on savings, saved in international banks, as per equation 39.
V. International Banking

This section is based on the work of Darity and FitzGerald (1982) and Vos (1994). In this literature, in contrast to the pure theory of capital movement where return differential determines the movement of capital, international banks are assumed to work on a segmented capital market, characterized by credit rationing. In order to capture this phenomenon, we begin by specifying the basic closure of this bloc (Eq. 40),

\[ \Delta B^{s+p} = \Delta J_{A} + \Delta J_{N} + \Delta J_{OPEC} + \Delta J_{A} - \Delta D_{N} \]  

Following Vos (1994), Eq. 40 equates the supply of funds from the three regions, net of Northern government demand for deficit financing, with the demand for credit by South, as well as the private sector of the North. Equations 39, 41 and 42 summarise the supply of funds. The credit rationing is effected as follows. First, priority will be given to Northern firms, as specified in Eq. 43.

\[ \Delta J_{N} = a_{41} + b_{41} J_{N} + c_{41} (i_{w} - r_{N})_{t-1} \]  

\[ \Delta J_{A} = a_{3} + b_{3} J_{A_{t-1}} \]  

Following Vos (1994), the supply of funds from the three regions, net of Northern government demand for deficit financing, with the demand for credit by South, as well as the private sector of the North. Equations 39, 41 and 42 summarise the supply of funds. The credit rationing is effected as follows. First, priority will be given to Northern firms, as specified in Eq. 43.

\[ \Delta B_{pN}^{d} = a_{43} + b_{43} J_{N} \]  

The remainder of funds are, then, directed to the South. The supply of credit to the South could, in principle, be specified using two approaches.

In Vos’ model, supplier’s perception of default is believed to be inferred from a critical outstanding stock of bank debt to export earning ratio (Vos, 1994: 218). From the borrower’s perspective Vos’ model essentially maintains that there is a certain debt management scheme, which is employed by Southern governments. Hence, the demand for bank loans in the South is limited by a maximum targeted level for the interest payment to foreign exchange earnings ratio. Thus, the ratio essentially implies the possibility of downward adjustment of demand for foreign inflows by Southern borrowers, based on their own assessment or debt management scheme, irrespective of supply. Here, the South’s demand for funds is not necessarily supply determined, and neither that demand is perfectly elastic.

7. See Vos (1994), chapter 4 in particular, for a historical analysis of the segmented nature of the global capital market.
8. This maximum/critical ratio not only limits borrowing but also indicates a high probability of default (Vos, 1994: 216).
On the other hand, the IMF’s MULTIMOD (Masson et al. 1990) follows quite a different formulation, comparing an interest to export ratio with a certain critical maximum, as an indicator of repayment capacity. Thus, the ratio basically determines supply, with demand by the South implicitly assumed to be perfectly elastic. Both suppliers and borrowers are assumed to use the same indicator. By ensuring that this ratio lies well below the target level (hence low probability of default), borrowers (in the South) may facilitate the possibility of an increase in the supply of funds to them.

Based on the experience of African countries which is characterized by absence of debt management and the existence of nearly perfectly elastic demand for external finance the IMF’s approach are likely to represent the more realistic set of assumptions. Hence, MULTIMOD’s approach is followed. Moreover, since the South represents an aggregate category within Vos’ (1994) model he does not detail a mechanism, which may be used to show how bank flows are allocated among borrowers in the South. Consequently, Vos’ model is extended here to incorporate the assumption that Africa represents a poorer risk category than the rest of the South and, hence, that it is only allocated surplus capital, once demand for such capital from the rest of the South has been met. The latter, in turn, will be determined by its historical maximum share, which is assumed to be exogenous within this model. There are two main reasons why this is likely to represent a realistic picture for African countries. Firstly, as documented by Eaton and Gersovitz (1980) and Odedokun (1996), bank flows to these countries are likely to be supply constrained. Secondly, Kasekende et al (1995) describe how such flows to Africa are characterized by a high level of supply volatility. Hence, these flows are given as in Eq.44.

\[
\Delta B_A = \Delta B_{N+p} - \Delta B_{p} - \Delta B_{OT} \tag{44}
\]

Where: \( \Delta B_A \leq \Delta B_A \), \( \Delta B_A \) is historically determined maximum

The second extension to Vos’ model relates to the allocation of funds across Africa. In this study the debt service ratio (Eq. 47) is used as the relevant indicator of both solvency and liquidity in the African context. This is consistent with the findings of Kasekende et al (1995) and other studies, which focus on short-term flows to Africa. Given the debt service ratio, the allocation mechanism developed in this study is based on the relative position of each African sub-region’s solvency indicator, to the target level for the same ratio set by suppliers. Thus, it is assumed that, although Africa is a worst risk category among Southern borrowers, allocation of inflows destined for Africa will depend on the relative level of the debt service ratio for each region. This is formally given as follows,

\[
\text{9. However, non-market restrictions on the flows of bank credit to Africa could be responsible for limiting the total level of inflows. Within the model, this is implemented by limiting the market-based flow to Africa to its historical maximum level.}
\]
\[ \Delta B_{Ai} = \eta_i \Delta B_A \]  

\[ \eta_i = \frac{(\lambda^*/\lambda_i)}{\sum_{i=1}^{n} \left( \frac{\lambda^*}{\lambda_i} \right)} \]  

\[ \lambda_i = \frac{i_w (1 + \mu_s) B_{Ai} + i_w F_A + Pr in_Ai + oDS_i}{X_{Ai}} \]  

Where: \( l^* \) is exogenously determined maximum debt service ratio (formulated by suppliers)

\( l_i \) Debt service ratio of the \( i \)th risk group (region) in Africa. The sum in \( \eta \) of Eq. 46 over \( n \) adds to 1.

Once this credit rationing to the South is effected, any gap between demand and supply in the global financial market is assumed to eventually clear for world interest rate, by way of equations 43 and 3.
VI. Identities Used

\[ \mathbf{X}_{Ai} = \mathbf{R}_{Ai} \mathbf{R}_i + \mathbf{R}_{S} \mathbf{R}_i + \mathbf{R}_{N} \mathbf{R}_i + \mathbf{X}_{n} \mathbf{R}_i \quad \text{Where } i = 1..4, \text{ the four commodities} \]  

[48]

\[ \mathbf{X}_{Ai} = \sum_{i=1}^{4} \mathbf{R}_{Ai} \mathbf{R}_i + \mathbf{tX}_{Ai} + \mathbf{P}_{oil} \mathbf{Xoil}_{Ai} \]  

[49]

\[ \mathbf{X}_{i}^{SS} = \mathbf{X}_{R_{Ai}} + \mathbf{X}_{R_{OT}} \]  

[50]

\[ \Delta J = \Delta J_{R} + \Delta J_{S} + \Delta J_{N} + \Delta J_{W} \]  

[51]

\[ \mathbf{Q}_{A} \mathbf{Q}_{A} = \mathbf{Q}_{R} \mathbf{Q}_{R} + \mathbf{Q}_{S} \mathbf{Q}_{S} + \mathbf{Q}_{N} \mathbf{Q}_{N} + \mathbf{Q}_{W} \mathbf{Q}_{W} \]  

[52]

The 52 equations above capture the main features of an African economy. Analytical solutions of this model for each of block of the model are offered in Alemayehu (2002). An empirical implementation of this theoretical set-up is now conducted and reported in subsequent publications of the TFED Division’s working paper series.
References


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Appendix 1

Country Classification and Test Statistics of Model Parameters

Appendix 1.1: Country Classification

1. North: Comprises OECD countries, World Bank definition including Greece and Portugal: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States. In terms of regional classification it is North America (USA and Canada), EEC, Australia, Japan and New Zealand.

2. South: All developing countries (World Bank, World Tables definition).

3. Other South: All developing countries excluding Africa (World Bank, World Tables definition)

3. OPEC: OPEC members excluding African and Western Hemisphere suppliers: Bahrain, Kuwait, Qatar, Saudi Arabia, United Arab Emirates (UAE) Iran and Iraq.

4. Africa*


Samples used for Econometric estimation of ESA model:

Botswana, Ethiopia, Kenya, Madagascar Malawi, Tanzania, Uganda, Zambia

4.2 North Africa (NA): ECA definition, excluding Sudan: Algeria, Egypt, Libya, Morocco and Tunisia.

Samples used for Econometric estimation of NA model: Algeria, Egypt, and Tunisia

4.3 West and Central Africa (WCA): ECA definition:
**West Africa:** Benin, Burkina Faso, Cape Verde, Cote d’Ivoire, Gambia, Ghana, Guinea, Guinea Bissau, Liberia, Mali, Mauritania, Niger, Nigeria, Senegal, Sierra Leone and Togo.

**Central Africa:** Burundi, Cameroon, Central African Republic, Chad, Congo, Equatorial Guinea, Gabon, Congo, Rwanda, Sao Tome and Principe and Zaire

**Samples used for Econometric estimation of WCA model:** Benin, Burkina Faso, Gabon, Ghana, Nigeria, Senegal, Sierra Leone, Cameroon, Central African Republic, Zaire.